Q AL Y N EXAM N EOME Y AND OPOLO Y S MME 201

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(1) Consider an annulus. Identify the antipodal points on the outer circle. Also identify the points on the inner circle that are 120 degree (i.e. $2\pi/3$) apart. Compute the homology and cohomology (with integer coefficients) of this quotient space obtained from the operations above.

(2) Let G be a finite graph

(a) Prove that $\pi_1(G)$ is a finitely generated free group

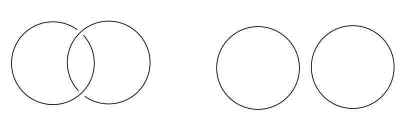
(b) Prove any finite index subgroup of a finitely generated free group is also a finitely generated free group.

(c) Prove that if F is a finitely generated free group and N is a nontrivial normal subgroup of infinite index, then N is not finitely generated.

(d) Show that a finitely generated free group has only a finite number of subgroups of a given finite index.

(3) Let Σ_g be the closed orientable surface of genus g. Prove that there is an n-sheeted covering $M_g \to M_h$ if and only if $g = n(h-1)+1$.

(4) Let A be the union of two once-linked circles in S^3; let B be the union of two unlinked circles in S^3, as shown in the picture. Show that the cohomology groups of $S^3 - A$ and $S^3 - B$ are isomorphic, but the cohomology rings are not.



A B